Identification of regular and chaotic isothermal trajectories of a shape memory oscillator using the 0-1 test

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Abstract

Shape memory oscillators are thermomechanical hysteretic systems that, in a wide range of model parameters, can exhibit complex non-periodic nonlinear dynamic responses under the excitation of a periodic force.

In this work the statistical 0-1 test based on the asymptotic properties of a Brownian motion chain is applied to periodic and non-periodic isothermal trajectories, to examine the type of motion.

The analysis is based on the computation of the control parameter K that approaches asymptotically 0 or 1 for regular and chaotic motions, respectively. The presented approach is independent of the integration procedure, being based on the characteristic sampling distance between the points of the analysed time series.

The numerical results show that the test is able to unambiguously distinguish periodic from chaotic trajectories.

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1 Introduction

Shape memory materials have a broad range of tecnhological applications covering various vibration problems like, for example, vibration isolation, reduction and attenuation; passive seismic protection via base isolators and dissipative bracings, as well as vibration actuators, thermally actuating switches and various types of sensors [1, 2].

The nonlinear dynamics of Shape Memory Oscillators (SMO) has been intensively studied in recent years [3, 4, 6, 7, 8, 9, 10, 11, 12, 13]. Shape memory materials exhibit a complex thermomechanical behavior and various approaches are possible for their constitutive modeling [1, 2, 5]. The study of the nonlinear dynamics of Shape Memory Oscillators has been undertaken essentially by means of two of them. On one hand, polynomial constitutive models inspired to Falk model [2] have been used in several works, like for example [3, 4, 11, 12]. This type of modeling has the advantage to permit a direct use of all the standard tools of smooth nonlinear dynamics, whereas it provides a simplified, indirect description of hysteresis at fixed constant temperature. On the other hand, fully thermomechanical models based on one or more internal variables associated with the fraction or the type of Martensite have been used in [6, 7, 13, 8]. This type of modeling offers an explicit description of hysteresis, as well as of the prediction of the temperature variations induced by the thermomechanical coupling, whereas it introduces the complications associated with the non-smoothness of the governing equations.

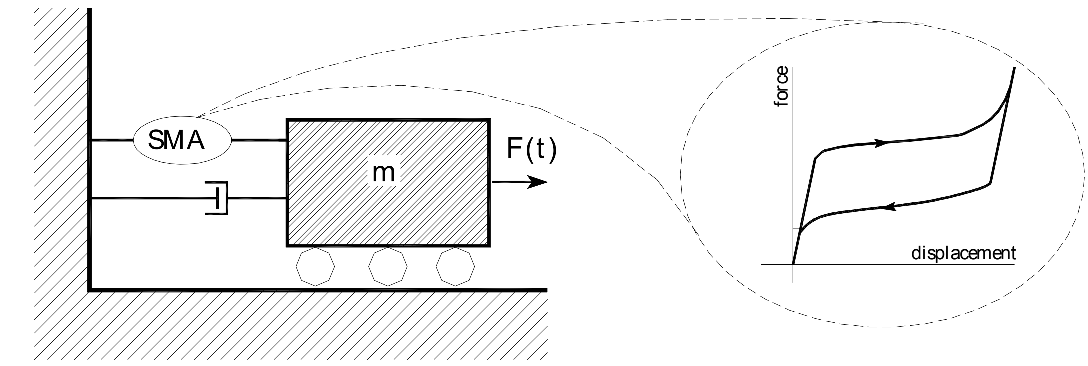
The occurrence of non-regular chaotic responses in SMO described via thermomechanical internal variable models has been observed and investigated not only with standard Poincar maps and Fourier spectra but also with a nonstandard tool of wandering trajectories [9, 10]. Note that, because of the increasing number of variables and system nonsmoothness, the identification of chaotic solutions by the maximal Lyapunov exponent becomes questionable. The present paper is continuation of the recent papers [9, 10] that are based on the model for SMO discussed in [6], and it advocates an application of the test 0-1 to distinguish regular and chaotic solutions [14].

2 Model and simulations

A SMO is a system composed of a main mass constrained by a Shape Memory Device (SMD), namely a suitable assembly of Shape Memory Materials that provides a pseudoelastic restoring force on the main structure (Fig. 1).

In general, SMDs are thermomechanical systems since the solid phase transformations occurring during mechanical loading involve the production/absorption of a certain amount of heat and this induces temperature changes that, in turn, significantly affect the mechanical response. As was shown in [6] a suitable constitutive framework for SMO is obtained if at each time t the state of the oscillator is described, besides displacement $x(t)$ and velocity $v(t)$, by an internal variable $\xi(t) \in [0, 1]$ that models the internal state of the SMD and by the temperature $\vartheta(t)$. In this work, as discussed below, the attention is focused on the isothermal response. To this end a simplified version of the constitutive model that describes a restoring force that exhibits isothermal pseudoelastic hysteresis loops is considered.

In the general modelling framework of [6], the effect of pseudoelastic shape memory



(a) (b)

Figure 1: (a) Schematic picture of a SMO model and (b) an example of a SMD hysteretic loop in the displacement-force plane.

devices on mass vibrations is considered within a thermomechanical environment characterized by a harmonic force excitation F and a convective rate of heat exchange Q

$$F = \gamma \cos \alpha t, \qquad Q = h\left(\vartheta_e - \vartheta\right), \tag{1}$$

where γ and α are the excitation amplitude and frequency, ϑ_e the fixed environment temperature and h the coefficient of convective exchange between the device and the environment (Fig. 1).

Modelling the pseudoelastic restoring force as in Ref. [6, 7] and expressing all the quantities in nondimensional form, the dynamics of the system is described by the following equations

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= -x + \operatorname{sgn}(x)\,\lambda\xi - \zeta v + F, \\ \dot{\xi} &= Z\left[\operatorname{sgn}(x)\,v - JQ\right], \\ \dot{\vartheta} &= ZL\left(\frac{\Lambda}{J\lambda} + \vartheta\right)\left[\operatorname{sgn}(x)\,v - JQ\right] + Q, \end{aligned} \tag{2}$$

where Λ is a constitutive function of ξ, ϑ whose explicit expression can be found in [6] and

$$Z = \frac{1}{\lambda + JL\vartheta + \dfrac{L}{\lambda}\Lambda + \dfrac{1}{\lambda}\dfrac{\partial \Lambda}{\partial \xi}}. \tag{3}$$

All system parameters and symbols used in the paper are presented in Tab. 1

Overall the full thermomechanical model depends on 7 material parameters that can be grouped as follows (see [8] for more details):

- *mechanical parameters* (q_1, q_2, q_3, λ) that reflect the basic features of the device (type and arrangement of the material) and determine the basic shape of the pseudoelastic loop observed in isothermal conditions;

- *thermal parameters* (L, h) that reflect the heat production, absorption and exchange with the environment and therefore determine the temperature variations of the device;

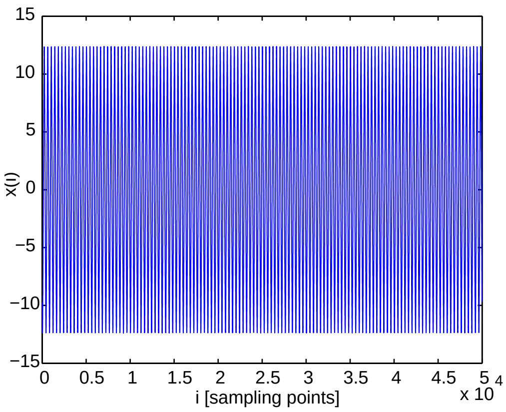
- a *thermo-mechanical* parameter J that determines the influence of the temperature variations on the pseudoelastic loops.

While temperature variations are very important in certain circumstances, as shown in Ref. [7], some of the aspects of the nonlinear dynamics of SMO can be appreciated even in a simplified setting in which temperature variations are neglected and therefore the response is assumed to be isothermal.

Taking this into account, henceforth the nonlinear dynamics of SMO will be studied under the constraint that $\dot{\vartheta} = 0$. This implies the thermal equilibrium of the SMD with environment $\vartheta = \vartheta_c$. The fully non-isothermal response will be investigated in further works. In such conditions the SMD is characterized simply by the 4 mechanical parameters (q_1, q_2, q_3, λ).

In order to test the applicability of the 0-1 test to detect chaotic responses, two trajectories are studied. The first one, called (a), exhibits regular-like behaviour while the second one, called (b), exhibits non-regular-like (or chaotic) behaviour (see Tab. 2).

The system has been integrated numerically over a time interval of 50000 excitation periods each contained 1000 points and then sampled that a rate of 4 points per the excitation period. This sampling constraint is related to the estimation of a minimum time lag by the Average Mutual Information criterion [15]. For a harmonic signal this time lag is equal to one quarter of the period [16]. The simulated time series are plotted in Figs. 2 a,b. Even at a first sight the two trajectories strongly differ by regularity. While Fig. 2a shows vibrations with constant amplitude, Fig. 2b exhibits non-regular oscillations with a fluctuating amplitude.

(a) 

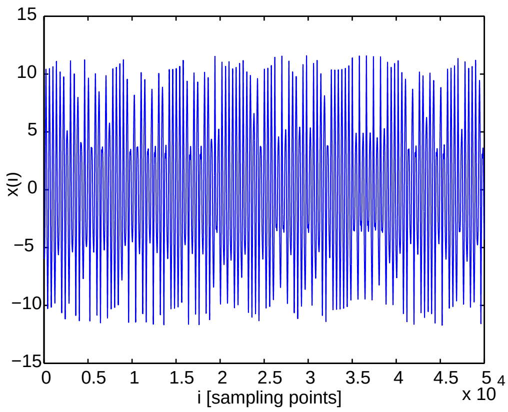
(b) 

Figure 2: Sampled displacement points x_i for two characteristic solutions (a)-regular and (b)-chaotic.

More hints about the dynamics of the system can be obtained by viewing the phase portraits where the difference between the regular and chaotic behaviour is clearly visible (Figs. 3a,b). In Fig. 3a, one can see a closed loop curve typical for a periodic system response while in Fig. 3b the lines form a complicated structure typical for a chaotic response. Additionally, the map obtained stroboscopically with one quarter of excitation period (Poincare-like-section) distinguishes the singular points (Fig. 3a) from a broad fractal-like distribution (Fig. 3b).

Table 1: List of parameters used in the paper

symbol	parameter
t	time
$x(t)$	displacement
x_i, i=1,2,3,...	sampled displacement points
$\overline{x}$, σ_x	average value and standard deviation of displacement
$M(n,c)$	total mean square displacement in new coordinates for n steps corresponding to sampled points
N	length of the sampled points in the displacement time series
$v(t)$	velocity
$p(i)$, $q(i)$	new coordinates obtained by nonlinear transformation
F	harmonic force excitation
γ	excitation amplitude
α	excitation frequency
Q	rate of heat exchange
ϑ_e	fixed environment temperature
ϑ	device temperature
h	coefficient of convective exchange between the device and the environment
$\xi \in [0,1]$	internal variable of Martensite fraction (pure Austenite $\xi = 0$, pure Martensite $\xi = 1$)
ξ_0	initial state of ξ variable
λ	denotes the transformation displacement factor maximum displacement that can be obtained by completely transforming the material from Austenite to Martensite
Λ	constitutive function of ξ and ϑ
sgn(.)	sign function
q_1, q_2, q_3	parameters responsible for modelling of a hysteretic loop, which control the slope of the upper loop plateau, position and slope of the lower loop plateau, respectively
J,Z	thermo-mechanical parameters
K_c and $\tilde{K}$	current (c dependent) parameter of the 0-1 test and its median average over 100 of c values.

The power spectra of the two time series are presented in Fig. 4 where the regular solution (Fig. 4a) shows 3 peaks representing the excitation frequency α and its consecutive odd multiple harmonics. On the other hand, the chaotic-like solution (Fig. 4b) apart of these, one can see a broadly distributed frequency band (0, 7α). While the analysis of the results shown in figures 3b and 4b could be already considered as a qualitative indication of chaotic vibration, in the next section a quantitative indication will be obtained by parametrizing the time series with a single number $\tilde{K}$ which can approach the values 0 or 1 depending on the motion regularity.

Table 2: Summary of system parameters used in simulations for time series a (regular-like response) and b (non-regular-like response), respectively.

	model parameters				**excitation** parameters	
Time series	λ	q_1	q_2	q_3	γ	α
(a)	8.125	0.98	1.2	1.017	1.0	0.400
(b)	8.125	0.98	1.2	1.017	1.0	0.227

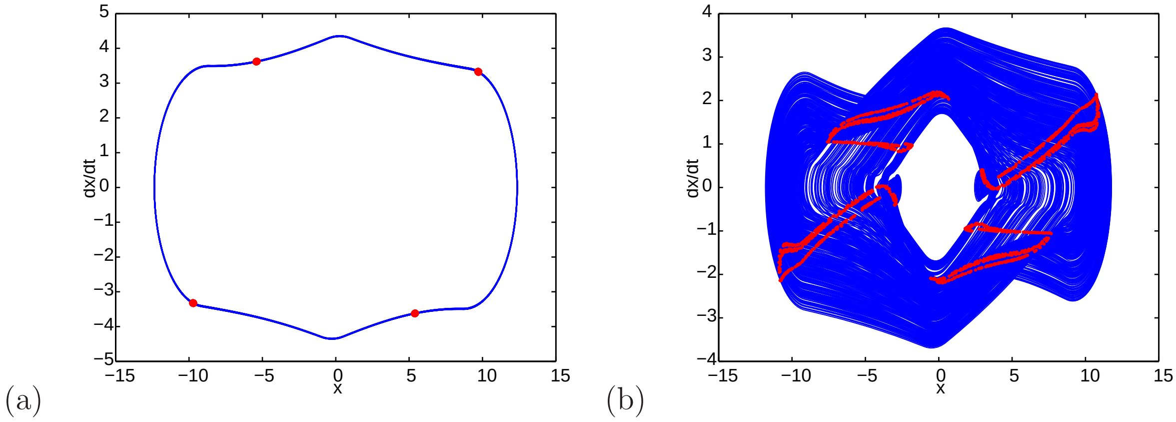


Figure 3: The phase portraits in $(\mathbf{x}, \dot{\mathbf{x}})$ with 4 points per period for two characteristic solutions (a)-regular and (b)-chaotic.

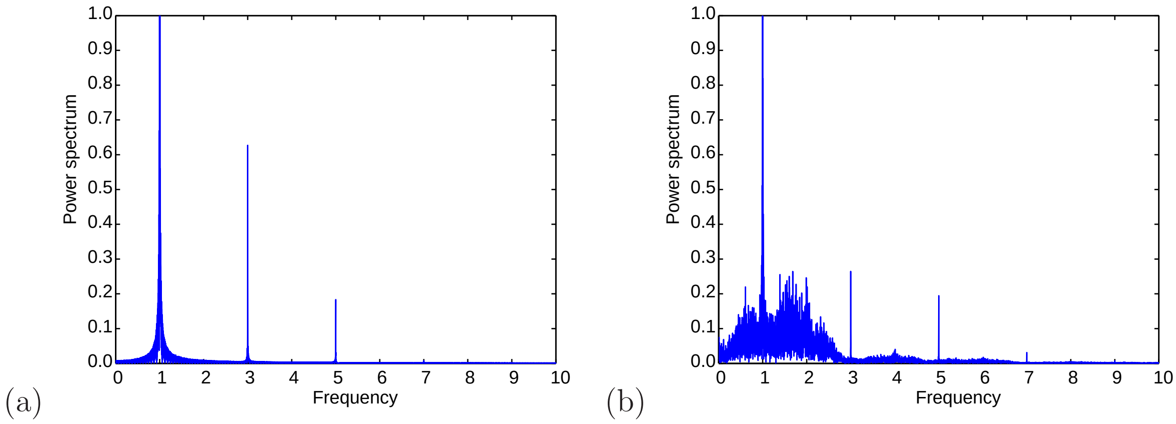


Figure 4: Power spectrum of the $\mathbf{x}$ coordinate for (a)-regular and (b)-chaotic solution expressed in the units of excitation frequency α.

3 Application of the test 0-1

The 0-1 test, proposed by Gottwald and Melbourne [14, 17] can be applied to any dynamical system of finite dimension to distinguish chaotic trajectories from regular ones. The method is based on the statistical properties of a single coordinate and, like spectral measures, is based on some universal properties of the dynamical system.

Starting from the displacement coordinate $x(i)$ of the sampled map (it could be any coordinate with defined sampling) the new coordinates $p(i)$ and $q(i)$ can be defined [15,

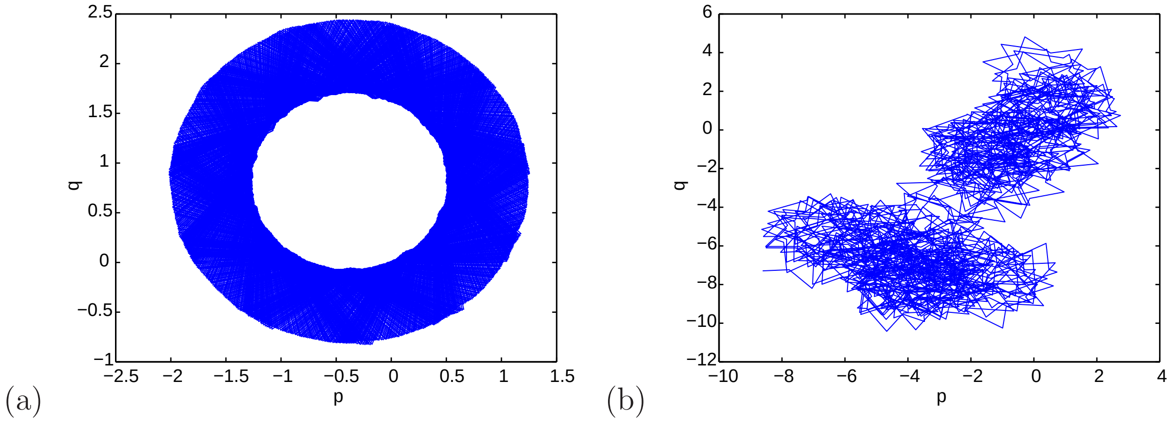


Figure 5: Phase portraits in new variables (p, q) for $i \in [1, N_{max}]$, $N_{max} = 2000$ and $c = 1.0$. (a) and (b) show the system evolution for regular and chaotic solutions.

16, 14, 17] as

$$p(i) = \sum_{j=0}^{i} \frac{(x_j - \overline{x})}{\sigma_x} \cos(jc), \qquad q(i) = \sum_{j=0}^{i} \frac{(x_j - \overline{x})}{\sigma_x} \sin(jc), \tag{4}$$

where $c \in (0, \pi)$ is a fixed frequency chosen arbitrarily, $q(i)$ is a complementary coordinate in the two dimensional space, and $\overline{x}$ is an average of x_i series. Note that starting from the bounded coordinate $x(i)$ one builds new series of $p(i)$ and $q(i)$ which can be either bounded or unbounded depending on dynamics of the examined process. In the following, $\overline{x}$ and σ_x denote respectively the mean value and square deviation of the examined x_i series.

In this way any chaotic vibration in the initial space $\mathbf{x}$ corresponds to an unbounded motion in (p, q) plane while a regular vibration (in the space $\mathbf{x}$) is related to a bounded motion in the (p, q) plane. To obtain a quantitative description of the examined solution, the method computes the asymptotic properties of the trajectory in the new plane, by means of the total mean square displacement

$$M(n, c) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \left[(p(j+n) - p(j))^2 + (q(j+n) - q(j))^2 \right]. \tag{5}$$

The asymptotic behaviour of $M(n, c)$ with growing n is strong only for some resonance frequency c (see the definitions of variables p,q in Eq. 4). To this end, in Ref. [18] the problems of averaging over c as well as sampling the data points are discussed extensively. The present investigation follows this approach as well as more recent papers by Krese and Govekar [19] and Litak et al. [20], which improve the convergence of the 0-1 test without the consideration of longer time series.

Consequently, the regression analysis [18, 19, 20] of the linear growth of $M(n, c)$ (Eq. 6) with increasing n is performed using the linear correlation coefficient which determines the value of the searching parameter K_c.

$$K_c = \frac{\mathrm{cov}(\mathbf{X}, \mathbf{M}(c))}{\sqrt{\mathrm{var}(\mathbf{X})\mathrm{var}(\mathbf{M}(c))}}, \tag{6}$$

where vectors $\mathbf{X}$=[1, 2, ..., n_{max}], and $\mathbf{M}(c)$= $[M(1, c), M(2, c), ..., M(n_{max}, c)]$.

The covariance $\mathrm{cov}(\mathbf{x},\mathbf{y})$ and variance $\mathrm{var}(\mathbf{x})$, for arbitrary vectors $\mathbf{x}$ and $\mathbf{y}$ of n_{max} elements, are defined as follows:

$$\begin{aligned} \mathrm{cov}(\mathbf{x},\mathbf{y}) &= \frac{1}{n_{max}} \sum_{n=1}^{n_{max}} (x_n - \overline{x})(y_n - \overline{y}), \\ \mathrm{var}(\mathbf{x}) &= \mathrm{cov}(\mathbf{x},\mathbf{x}). \end{aligned} \tag{7}$$

Finally, the median is taken of K_c-values (Eq. 9) corresponding to 100 different, systematically chosen values of $c \in (0,\pi)$.

In practical applications [18, 19, 20] the infinite limits is terminated to N_{max} and n_{max}, respectively ($n_{max} < N_{max}$). In the current calculations N_{max}=2000 and n_{max}=200 were assumed. For the fixed value $c = 1.0$, Fig. 4 shows the phase portrait in the $p-q$ plane for regular (a) and chaotic (b) solutions. It is to remark that Fig. 4a indicates bounded while Fig. 4b unbounded "walk" (see also the difference in scale).

To exclude the influence of the resonance frequency c, the final results as the median $\tilde{K}$ were chosen from all K_c calculated for a hundred values of c chosen systematically from the interval $[0,\pi]$. In this case, the median values of $\tilde{K} = -0.008 \approx 0.0$ and $K = 0.88 \approx 1.0$ were estimated for regular and chaotic cases, respectively.

4 Summary and conclusions

A quantitative analysis of the regularity of the dynamical solutions of a shape memory alloy oscillator in isothermal conditions has been carried out by means of the 0-1 test. The results show that regular and chaotic responses can be easily distinguished by the method. The presented approach gives a quantitative criterion for chaos similar to the maximum Lyapunov exponent, which estimation faces some difficulties when it comes to non-continuous and hysteretic systems [21]. In this context, the 0-1 test is very useful. Other advantages include a low computational effort and the possibility of its application in real time.

As demonstrated by Falconer et al. [18] the method can be also used to experimental data. Furthermore, it has been also shown that the 0-1 test could be applied to dynamical systems with additive noise and a good signal to noise ratio [17].

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