Meccanica **39:** 105–112, 2004.

Nonlinear Analysis of Experimental Time Series of a Straight Turning Process

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(Received: 2 October 2002; accepted in revised form: 9 May 2003)

Abstract. We investigate vibrations generated in a straight turning process. Applying correlation functions and the Fourier transform to experimental time series, we have analysed their nature. Particularly, we have identified whipping caused by the non-ideal suspension of the cutting workpiece. Our investigation shows also a large stochastic component of vibrations. It can be the effect of random forcing due to the initial roughness of a cutting surface and/or spontaneous chips breaking.

Key words: Nonlinear vibrations, Turning process, Cutting process, Correlation function, Whipping.

1. Introduction

Recently, industry needs for products of technological processes, including cutting and turning, have been increasing rapidly. These needs as well as global trade forced the improvement of technological processes to satisfy the market in terms of quantity, quality and price. A big progress has already been made in tool developments and in process real time monitoring and control [7, 17]. Nevertheless, the problem of obtaining a high quality final product during massive production has not been solved yet. So the next step is to identify and understand the basic physical phenomena that take place during the cutting or turning processes. These processes include such peculiar nonlinear ones as dry friction [22] or impacts after tool-workpiece contact loss [10]. For this, theoretical and experimental works on cutting and turning processes have been focused on identification of quasi-periodic chatter vibration sources [12]. Other instabilities of cutting process include stochastic and chaotic vibrations [5, 8, 11, 15, 19]. All these vibrations, appearing during the cutting process, are harmful for machining technology. They lead to deviations in final surface parameters.

Preliminary studies on nonlinear modelling and the existence of primary and secondary chatter were conducted by Grabec [2, 3], Marui *et al.* [12], Gradisek *et al.* [4], Wiercigroch and Cheng [23], Warmiński *et al.* [20, 21], Litak *et al.* [9], Pratt and Nayfeh [14], Stepan and Kalmar-Nagy [18].

The corresponding experimental results, mostly on an orthogonal cutting process, have been discussed in several recent papers [11, 13, 19]. Here we continue in this direction and examine the straight turning process and identify the nature of generated vibrations.

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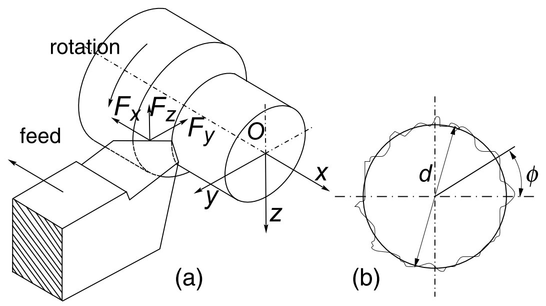

Figure 1. The experimental standing of a straight turning process (a). Schematic picture of yz projection of turned workpiece in presence of vibrations (b). Note the deformation of surface as a function of angle ϕ.

2. Experimental Standing

We start by defining the geometry of process and the description of measured quantities. Our experimental standing for a straight turning process consists of the tool–workpiece system (Figure 1(a)). The necessary coordination systems include cutting forces in x, y, z directions: F_x, F_y, F_z. Forces were measured by a quartz dynanometer. The electrical charges delivered there from the quartz sensors are converted by charge amplifiers into proportional voltages, which may be processed by the usual instruments. The displacements of a workpiece, in orthogonal directions, y, z were measured by laser displacement-meter devices placed behind and above the workpiece, respectively. These displacements were related to the absolute position of a workpiece during the turning process.

In Figure 1(b) we present a schematic plot of the final product surface with some considerable roughness. Folds on the surface are caused by harmless vibrations generated during the technological process. The aim of this paper is to perform mathematical analysis of the experimental time series collected by using the above equipment.

3. Experimental Results and Analysis

We have performed a series of experiments changing the technological parameters like a rotational velocity Ω and a diameter of the workpiece d (Figure 1(b)) and the assumed cutting depth h. The tests were also done for different kinds of cutting materials. Here we present the results for cutting a workpiece of gray iron with the following process parameters: $d = 33$ mm and $\Omega = 770$ rev/min and a feed velocity $v_x = 0.1$ mm/rev, $h = 1$ mm. The frequency of data accounting was fixed at 2000 scans per second.

The results for forces measured during the turning are plotted in Figure 2. As we expected, the largest component of average force was in z direction (cutting force) while the smallest one was in x direction. ($\overline{F_x} = 134$ N, $\overline{F_y} = 172$ N, $\overline{F_z} = 232$ N). The time histories of these components are presented in Figure 2. Note that oscillation amplitudes for all components are quite large. The amplitude reaches even 80% of the average values. In case of F_y, one can also note the change of sign (Figure 2(b)) for $t \approx 0.4$ s. This may be due to the friction phenomenon between the tool and a chip [22].

Similar behaviour is also typical for displacements. In Figure 3 we present time histories of workpiece y and z displacements, respectively. Now we are going to explain such behaviour.

Firstly, one should note that vibrations are modulated with slow component of about 13 periods per second. This period can be directly related to the rotational frequency of work-

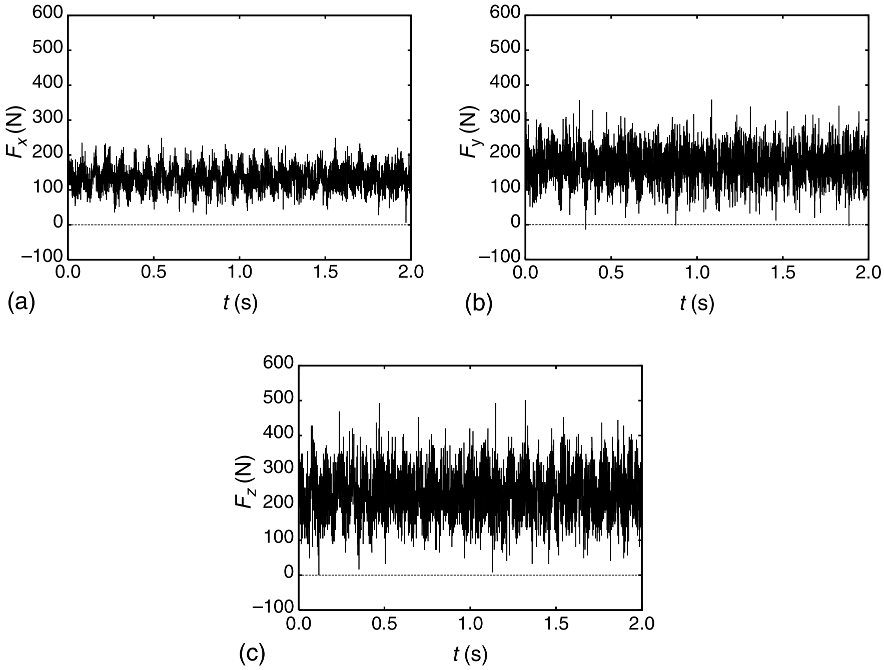

Figure 2. Time series of cutting forces: F_x (a), F_y (b), F_z (c).

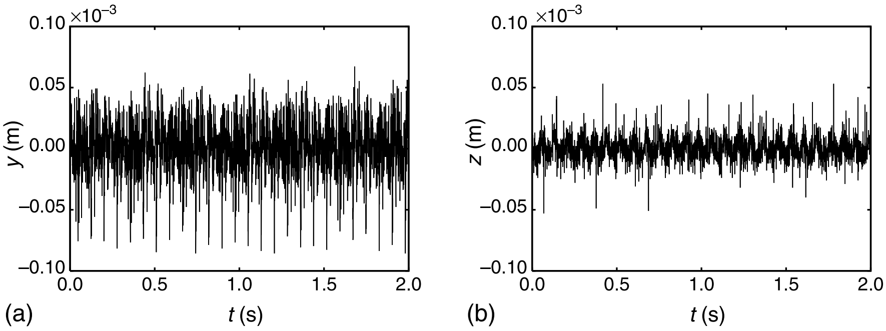

Figure 3. Time series of displacements: y (a), z (b).

piece revolution Ω. Thus, the natural explanation of this modulation is whipping due to an unbalanced workpiece. This effect is always present in the cutting process.

Secondly, all time history plots in Figures 2 and 3 indicate that there is a large component of non-periodic vibrations. Clearly, there are two possibilities [1, 6, 16]: either the vibrations are chaotic (in the sense of deterministic chaos) or the system vibrates stochastically with external and internal random influences. To examine this effect further we calculate the autocorrelation function $C(n)$ for the series of experimental points

$$\tilde{s}(m\tau) = s(m\tau) - \bar{s}, \tag{1}$$

where s denotes chosen measured quantity (i.e. force components F_x, F_y, F_z or displacements y, z), τ is the time interval between measurements, m is a natural number or 0 and denotes the number of measurement ($t = m\tau$ is a discrete measurements time), and $\bar{s}$ is an average value

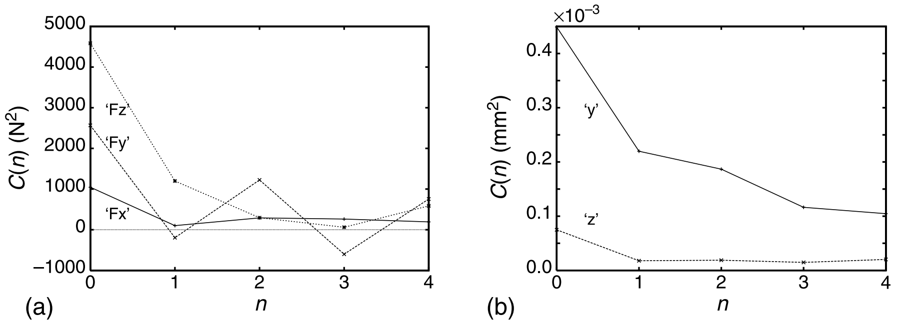

Figure 4. Autocorrelation function $C(n)$ for cutting force components F_x, F_y, F_z (a) as well as the displacements y, z (b).

of $s(m\tau)$. The autocorrelation function can be written as follows:

$$C(n) = \lim_{N \to \infty} \frac{1}{N} \sum_{m=0}^{N} \tilde{s}(m\tau) \times \tilde{s}((m+n)\tau). \tag{2}$$

In our case, N denotes a number of measurement events chosen to be $N = 4000$.

This function $C(n)$ was calculated for cutting force components F_x, F_y, F_z and displacements y, z, showed in Figures 4(a) and (b), respectively. It was plotted against in the region of small 'correlation time distance' $n(n = 1, \ldots, 4)$. It is easy to note that in cases of F_x, F_z (Figure 4(a)) and x (Figure 3(b)) the following relation is fulfilled:

$$C(\tau) < \frac{C(0)}{2}, \tag{3}$$

showing that the correlation time is very small. That is, it is of the order of a single time step τ. Interestingly, such relation indicates that the autocorrelation function can be simplified to

$$C(n) = C(0)\delta_{0n} \quad \text{for small } n, \tag{4}$$

where δ_{0n} is a Kronecker delta, corresponding rather to the chaotic signal [16] where the close trajectories are divergent. However, in other cases for F_y and y (Figures 2 and 3), the behaviour is different from those described by equation 2, showing evidently some higher frequency components.

In Figure 5 we have plotted all experimental points using a three dimensional coordinate system of F_x, F_y, F_z. Each point $\mathbf{F}_n$ has been accounted using a discretized time $t_n = n\tau$:

$$\mathbf{F}_n = \mathbf{F}(n\tau) = [F_x(n\tau), F_y(n\tau), F_z(n\tau)]. \tag{5}$$

The probability distribution of experimental points $P(\mathbf{F})$ defined for small force cubes ΔF_x $\Delta F_y \Delta F_y$ appeared to have a continuous character.

For better clarity we present, in Figures 6(a)–(c), the power spectra, in the logarithmic scale, for force components (F_i, $i = x$, y and z, respectively):

$$C(\omega') = FT(C(t)) = \frac{1}{N} \sum_{k=1}^{N} C(t)\, e^{\mathrm{i}n\tau\omega'} \tag{6}$$

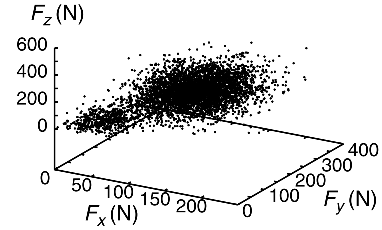

Figure 5. Experimentally measured points in a coordinate space spanned by cutting force orthogonal directions (F_x, F_y, F_z). Note that the distribution is continuous.

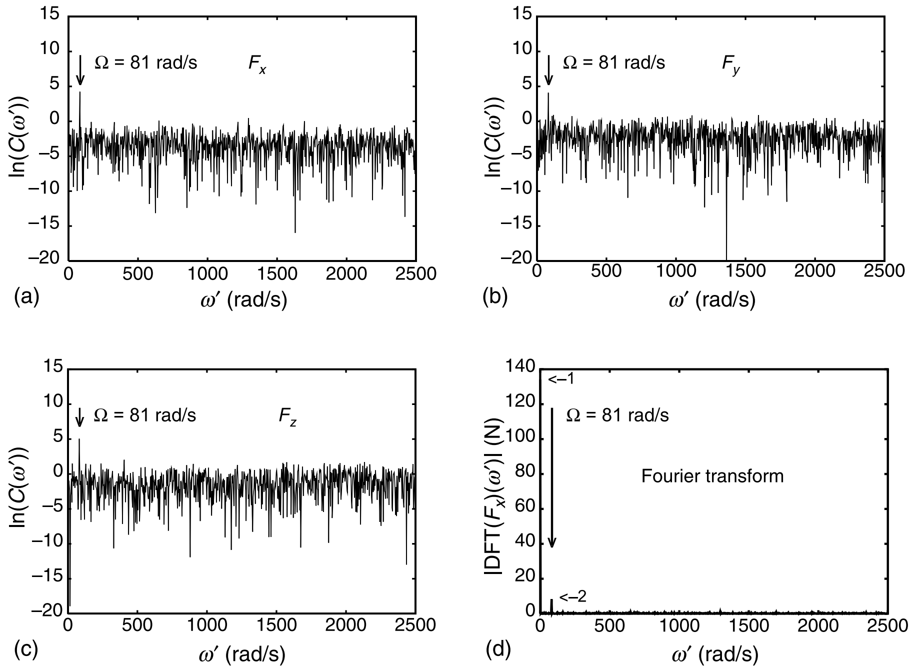

Figure 6. Power spectra of cutting forces in the logarithmic scale: $\log P_{F_i}(\omega') = \log(C(\omega'))$ for $F_i = F_x$ (a), $F_i = F_y$ (b), $F_i = F_z$. (c) and a Fourier Transform $\mathrm{FT}F_x(t)$. Short arrows (d) show the constant static term $F_x(\omega' = 0)$ (1) and the amplitude $F_x(\omega' = \Omega)$ (2).

for large enough N. Similarly, a Fourier transform for F_x component (Figure 6(d)) reads

$$F_x(\omega') = \frac{1}{N} \sum_{k=1}^{N} F_x(t)\, e^{\mathrm{i}n\tau\omega'}. \tag{7}$$

Each of the power spectrum presented in Figures 6(a)–(c) is rich in frequencies; however, one can easily find ω', which is the dominant one. It corresponds to a rotational velocity of the workpiece $\Omega \approx 81$ rad/s. Thus, we conclude that the unbalanced workpiece shaft is causing a whipping effect. It acts like additional parametric forcing, which introduces excitation energy to the system. This effect is also visible in Figure 6(d). Surprisingly, plotted in the normal scale, the amplitude $F_x(\omega' = \Omega)$ is much smaller than $F_x(\omega' = 0) \approx \overline{F_x}$. Note, in Figure 6(d), we introduced short horizontal arrows to distinguish the static term level $F_x(\omega' = 0)$

(1) and the amplitude related to the forcing frequency Ω $F_x(\omega' = \Omega)$
(2) We have also found a similar tendency for F_y and F_z components.

This continuous distribution of experimental points, together with the rather non-typical behaviour of the correlation function, lead us to the conclusion that the system shows regular vibrations disturbed by stochastic influences [1, 6]. To probe the possibilities of chaotic and stochastic motions, we conducted calculations of correlation length l [16]. Usually in the case of regular vibrations, the stochastic component smears the trajectory inside a circle of radius l around each point of a system trajectory [1]. To examine such smearing, one look for a proper configuration space and its dimension. As we have measured three components of forces $\mathbf{F}$, there is no need to consider an embedding dimension as an extraction from one of the Force components, as has been done in Marghitu *et al.* [11]. Thus instead of detailed embedding space analysis, we introduce rather a coordination space spanned by the cutting force orthogonal directions (F_x, F_y, F_z) and assume that the characteristic embedding dimension is fixed at 3. Now we can calculate the 'spatial' correlation function, in a force space [F_x, F_y, F_z]:

$$\tilde{C}(l) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{ij} \Theta(l - |\mathbf{F}_i - \mathbf{F}_j|), \tag{8}$$

where $\Theta(x)$ is the Heaviside step function and the correlation dimension D_2 for small enough l:

$$D_2 = \frac{\log \tilde{C}(l)}{\log l/l_0} + \text{const}. \tag{9}$$

Figure 7 shows the results of such calculations. Here, the slope of $\log \tilde{C}(l)$ against $\log l/l_0$ (l_0 is an arbitrary length which was taken to be 1 N for a practical reason) for a linear part of a plotted curve (in a small l region) gives the result for a correlation dimension D_2 which is defined as $\tan\alpha \approx 3$ (Figure 7). This, a purely integer value of D_2, ends all speculations on the chaotic origin of vibrations and provides a final argument for the random nature of vibrations in our straight turning process. A large characteristic correlation length $l_c \approx 90$ N (Note $l_c = 90$ N corresponds to $\log(lc/l_0) = 4.5$ in Figure 7) indicates that the stochastic component is considerably large.

The final surface obtained in the process is presented in Figure 8. Note, the deformations of surface from the flatness were plotted against the angle ϕ defined earlier in the schematic picture – Figure 1(b). From the stochastic vibrations we find that the discrepancies from the roughness line reach about 5% of the assumed cutting depth $h = 1$ mm.

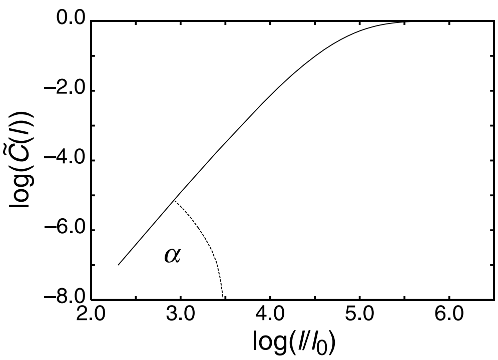

Figure 7. The correlation dimension D_2 for small enough l.

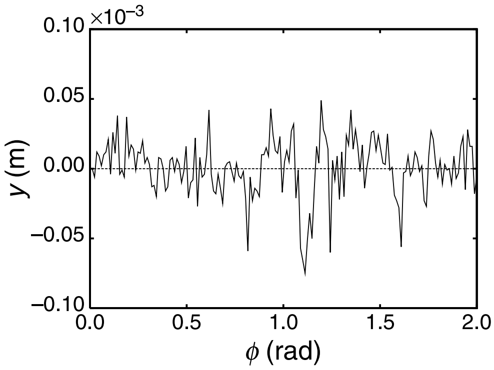

Figure 8. Final surface deformation as a function of angle ϕ estimated from measured y displacements of a workpiece during cutting process. The angle ϕ was defined in schematic picture Figure 1(b).

4. Summary and Discussion

The results obtained from our samples show the parametric whipping excitation and most of all the large stochastic component of vibrations. We have used standard mathematical methods like correlation function approaches to analyse the outgoing experimental signal. On the basis of experimentally measured forces, their probability distribution and spatial correlation functions, we have convinced ourselves that the stochasticity has a random noise character. The characteristic force correlation function length $\log(l_c/l_0) \approx 4.5$ (Figure 7) relates to the size of the distribution $l_c = |\mathbf{F}_c| = 90\,\mathrm{N}$ (Figure 5). The above conclusion is particularly supported by the fact that our workpieces of gray iron have been characterised by the typical roughness of initial surfaces [8] or the dynamics of chips breaking, causing external random forcing and grain composition of the material giving internal randomness during the technological process. Our results justify those theoretical approaches to a cutting process, which assume the effects of random noise in orthogonal cutting [5, 15, 23] and, more recently, straight turning [10] processes.

Interestingly, our preliminary investigations showed that the effect of random noise was smaller, but it is not eliminated in workpieces with other kinds of material.

Acknowledgements

The authors are very grateful to Prof. J. Warmiński and Dr. K. Zaleski for helpful discussions. The paper was supported in part by KBN grant No. 126/E-361/SPUB/COST/T-7/D2 42/99.

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