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Chaotic vibrations in a regenerative cutting process

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Abstract

We have analysed vibrations generated in an orthogonal cutting process. Using a simple one degree of freedom model of the regenerative cutting, we have observed the complex behaviour of the system. In presence of a shaped cutting surface, the nonlinear interaction between the tool and a workpiece leads the to chatter vibrations of periodic, quasi-periodic or chaotic type depending on system parameters. To describe the profile of the surface machined by the first pass we used a harmonic function. We analysed the impact phenomenon between the tool and a workpiece after their contact loss. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

The stability of a cutting process influences directly the quality of a final surface. The control of process in various working conditions is an important problem for machining technology. Instabilities of process usually manifest as harmful chatter vibrations generated during the cutting. Recently, many papers tackled that problem focusing on the conditions of appearing such vibrations. Apart from regular (periodic or quasi periodic) vibrations, possibilities of chaotic ones were also investigated theoretically [1–10] as well as experimentally [5,11,12]. The aim of this paper is to explore the mechanism of the chatter vibration generation using a simple model of a cutting process with one degree of freedom. In this paper, we will clarify the role of impacts after tool–workpiece contact loss.

2. The model of a cutting process

The one degree of freedom physical model of the orthogonal cutting process is presented in Fig. 1. Here k denotes the effective spring of a workpiece, c is the damping coefficient, h_0 is the assumed cutting depth while h the actual one, Ω_0 is a rotational velocity and v_0 is a relative velocity between the tool and the workpiece tangent to the workpiece surface ($v_0 = \Omega_0 r$, where r is a radius of the workpiece). The horizontal displacement of the workpiece symmetry axis at the time t is denoted by y. After the first pass of tool the actual cutting depth $h(t)$ can be expressed as

$$h(t) = h_0 - y(t) + y(t - T), \tag{1}$$

where the $y(t-T)$ corresponds to the position of the workpiece of during the previous pass, T is the period of revolution. In our approach, assuming constant relative velocity $v_0 =$ constant, the actual cutting of depth $h(t)$ will be determined by the dynamics of the model [10]:

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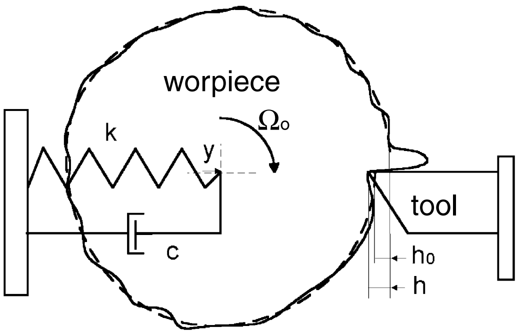

Fig. 1. The model of an orthogonal turning process. The full line illustrates the workpiece surface cut in the presence of chatter vibrations while the dashed one corresponds to the smooth surface cut in stable conditions.

$$\ddot{h} + 2n\dot{h} + p^2 h = -\frac{1}{m}\mathrm{sgn}(v_0 + \dot{h})F_y(h), \tag{2}$$

where p is a frequency of free vibrations and $2n = c/m$ is a dimensionless damping coefficient. Finally, F_y is the thrust force, a horizontal component of a nonlinear cutting force, m is the effective mass of a workpiece. The above equation (Eq. (2)) can be written in terms of current position $y(t)$ (Eq. (1)) corresponding to the actual position of the workpiece as

$$\ddot{y}(t) + 2n\dot{y}(t) + p^2 y(t) = \frac{1}{m}\Big(\mathrm{sgn}(v_0 - \dot{y}(t))F_y(h) - \mathrm{sgn}(v_0 - \dot{y}(t))F_y(h_0)\Big). \tag{3}$$

The thrust force F_y is mainly based on a dry friction part (between the tool and a chip) with power law dependence on the actual cutting depth h [10]:

$$F_y(h) = \Theta(h)Kw(h)^{3/4}, \tag{4}$$

where K denotes the cutting resistance, w is a chip width and Θ is the Heaviside step function. To go further in our analysis, we have assumed in the first approximation, that $y(t-T)$ can be described by a periodic function

$$y(t-T) = a\cos(\omega t - \phi) = a\cos\left(\frac{\omega x}{v_0} - \phi\right), \tag{5}$$

where ω, a and ϕ are the frequency, amplitude and phase of a surface shape modulation, respectively. $x(t)$ denotes the relative distance passed by the tool on the cylindrical surface of a rotating workpiece. Note that our model (Fig. 1) includes also the contact loss between the tool and a workpiece. Such phenomenon can be particularly important during the cutting process with a high speed. Here we assume the restitution parameter $\beta \leqslant 1$ connected with the impact after contact loss.

$$\dot{y}(t^+) = -\beta\dot{y}(t^-), \tag{6}$$

where t^+ and t^- denotes time before and after impacts, respectively. The above assumptions (Eqs. (5) and (6)) will be crucial in our investigations in the following section where we discuss the results of simulations.

3. Numerical simulations

Eq. (2) has been solved numerically for the realistic system parameters [5,6,9]: $K = 1.25 \times 10^9$ N/m^2, $w = 3.0 \times 10^{-3}$ m, $h_0 = 0.3 \times 10^{-3}$ m, $p = 816$ rad/s, $m = 17.2$ kg, $a = 0.2 \times 10^{-3}$ m, $\phi = \pi/2$, $\beta = 0.75$ and a relatively small damping $n = 4.3$ s^{-1}. In Figs. 2(a)–(d) we show time histories of cutting depth $h(t)$ for various frequency. The negative value of h means that the tool lost a contact with the workpiece. Fig. 2(a) presents the results for cutting with 'parametric excitation' (Eq. (5)) $\omega = 800$ rad/s. Note that, this case corresponds to a relatively small velocity v_0 (but still $v_0 > \dot{y}$). One can see that the cutting process is very smooth with small fluctuations around assumed h_0. In contrast to that Fig. 2(b) (for a larger frequency $\omega = 1200$ rad/s) shows unstable cutting with large fluctuation of a

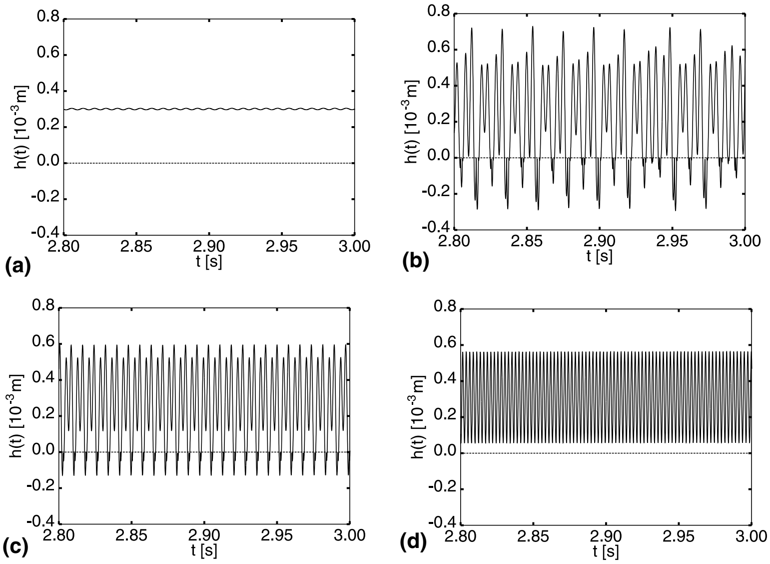

Fig. 2. Time histories of workpiece motion $y(t)$ for various frequency ω: (a) $\omega = 800$ rad/s, (b) $\omega = 1200$ rad/s, (c) $\omega = 1600$ rad/s, (d) $\omega = 2600$ rad/s.

cutting depth h. The time history $h(t)$ manifests the contact loss phenomenon. Moreover, vibrations generated in the system look aperiodic. It could be caused by the lack of synchronisation between impacts incidents and the driving frequency ω. Thus, the aperiodic part is appearing after the escape of tool into the region of negative $h < 0$ (Fig. 2(b)). On the other hand, the same system but $h > 0$ seems to be more regular. Such behaviour is the benchmark of an intermittent mechanism of chaotic motion [13]. Surely, as in other mechanical examples [14], it appears due to impacts. Increasing the ω ($\omega = 1600$ rad/s in Fig. 2(c)) we transit back to regular vibrations of cutting depth $h(t)$. Here, in spite of a large amplitude and a tool–workpiece contact losses the impacts are synchronised with a 'parametric forcing' ω. Finally, for a relatively large ω ($\omega = 2600$ rad/s in Fig. 2(c)) which corresponds to cutting with a high speed v_0, the vibration amplitude slightly decreases.

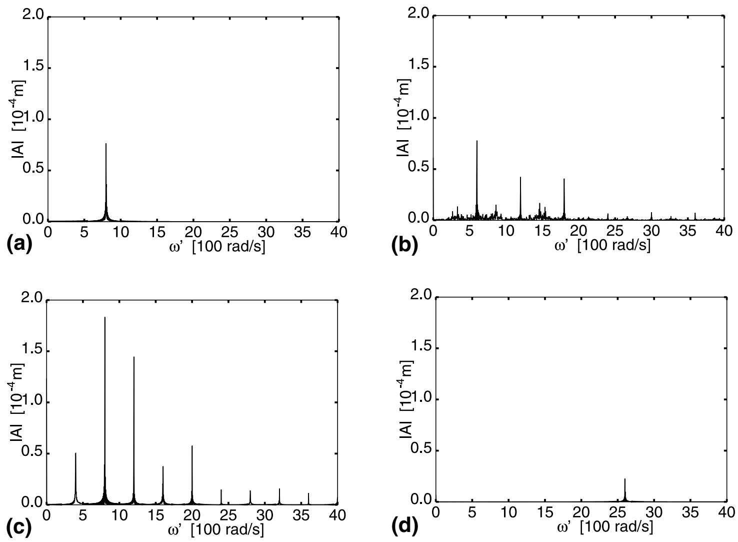

Fig. 3. The amplitude of workpiece vibrations y versus frequency ω'. The system parameters have been chosen as in Fig. 2.

To explore the vibrations of the system, we have done the Fourier transform for each of above cases. Figs. 3(a)–(d) present the amplitude of workpiece vibrations $y(t)$ versus frequency ω' with the system parameters as in Fig. 2. Figs. 3(a) and (d) show spectra with a single frequency corresponding to driving one ω indicating on synchronized motion of a workpiece. Fig. 3(b) has a combined spectrum of singular frequencies and continues interval of ω'. This is a typical output of chaotic systems. In Fig. 3(c), one can find a lot of characteristic frequencies ω' which are represented by natural magnification of $\omega/4$ ($4\omega'/\omega = l$, where l is a natural number).

Interestingly, for an small driving frequency $\omega = 800$ rad/s, which corresponds to a small rotational velocity of a workpiece, fluctuations of cutting depth h are small Fig. 2(a). Physically, in this case, the workpiece in its motion y can follow the driving 'parametric forcing' by the initial shape modulation $y(t - T)$. This is resulting in relatively large vibration amplitude of the workpiece (Fig. 3(a)). For a large driving frequency $\omega = 2600$ rad/s, we obtain the opposite situation. Now fluctuations of h have larger amplitude (Fig. 2(d)) while vibrations y smaller one (Fig. 3(d)). In this case a workpiece, due to its large inertia, cannot follow the changes of the initial shape forcing the large variations of h via Eq. (1).

4. Summary and conclusions

We have considered vibrations of a tool-workpiece system in an orthogonal turning process. The simple one degree of freedom model we used includes the basic phenomena as friction between a chip and the tool, nonlinear-power low character of the cutting force expression as well as the possibility of a contact loss between the tool and the workpiece. In our model we observe the complex behaviour of the system. In presence of a shaped initially cut surface, the nonlinear interaction between the tool and a workpiece leads the to chatter vibrations of a periodic, quasi-periodic or chaotic type. The main role in chaotic motion is played by impacts generated after the a tool-workpiece contact losses. Clearly, they enable an intermittent transition from a regular to chaotic system behaviour. In spite of the fact that our results are obtained by very simple model, we believe that we found the important mechanism of cutting instabilities by an impact phenomenon.

Acknowledgements

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